Optimal fertility under age-dependant labor productivity

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Alfred Sauvy on fertility

- In the so-called *Rapport Sauvy* (1962), the French demographer Alfred Sauvy argued that Wallonia's fertility was too low.
- Sauvy recommended a 20 % rise in fertility.
- Main intuition:
  - Low fertility leads to a low productive economy composed of old workers with old ideas living in old houses.
- Population ageing would thus be bad for productivity and growth.

Alfred Sauvy on "optimal fertility"

- Sauvy's diagnosis claims that Wallonia's observed fertility was *lower* than the socially optimal fertility.
- But "socially optimal" in which sense?
- In his *Théorie Générale de la Population*, Sauvy argued that lots of social objectives exist to define what the "optimal fertility" is.
    - Total welfare, average welfare, total quantity of life, total population size, culture and knowledge, longevity and health, power...
- Can we reconcile Sauvy's diagnosis with an economic theory of optimal fertility?

This paper

- Our goal is to incorporate Sauvy's views on optimal fertility into an economic model of optimal fertility.
- We extend Samuelson's (1975) into a 4-period OLG model with 2 active periods, allowing for some decay in old workers' skills.
- We derive the fertility rate maximizing long-run average welfare.
- We then calibrate the model to see whether Sauvy's diagnosis can be rationalized.

Our results

- We show that the optimal fertility rate equalizes, at the margin:
  - the sum of the capital dilution effect (Solow effect) and the labor age-composition effect (Sauvy effect)
  - the intergenerational redistribution effect (Samuelson effect)
- In theory, it is possible to incorporate Sauvy's views within Samuelson's framework.
  - A higher decay of old workers skills can favor a higher optimal fertility.
- But at the quantitative level, simulations suggest that it is extremely difficult to regard Wallonia's TFR as suboptimal.

Outline

1. The model
2. The long-run social optimum
    1. The social planner's problem
    2. The optimal fertility rate
3. Numerical illustrations
4. Conclusions

The model: demography

- 4-period OLG model:
  - period 1: childhood (no work);
  - period 2: work, consume, save and have n children;
  - period 3: work, consume and save;
  - period 4: retirement.
- Lifetime is risky: only a fraction π of each cohort reaches the old age.
- The population size follows the dynamic law:

$$N_{t+1} = nN_t$$

where N_t denotes the number of individuals born at t.

The model: labour force

- The total labour force at time t, denoted by L_t, is equal to:

$$L_t = N_{t-1} + \alpha N_{t-2}$$

  where $\alpha \in [0, 1]$ captures the extent of decay in the skills of old workers.

- This gap in productivity between the young and the old may be due to various causes.
  - Boucekkine et al (2002): the education of old workers dates back to a more distant epoch, which can make their skills relatively out of date.

- Using the law for population, total labour can be rewritten as:

$$L_t = N_{t-2}\,(n + \alpha)$$

The model: production

- The production of an output Y_t involves capital K_t and labor L_t:

$$Y_t = F(K_t, L_t) = \bar{F}(K_t, L_t) + (1-\delta)K_t$$

where δ is the depreciation rate of capital. $\bar{F}(K_t, L_t)$ has CRS.

- The production process can be rewritten as:

$$y_t = F\left(k_t, \frac{N_{t-2}(n+\alpha)}{N_{t-2}n}\right) = F\left(k_t, 1+\frac{\alpha}{n}\right)$$

where $y_t \equiv \frac{Y_t}{N_{t-1}}$ and $k_t \equiv \frac{K_t}{N_{t-1}}$.

The model: preferences

- Individuals have preferences represented by:

$$u(c_t) + \beta u(d_{t+1}) + \pi\beta^2 u(b_{t+2})$$

where c_t, d_t and b_t denote consumption at period 2, 3 and 4 of life.

- As usual, $u'(\cdot) > 0$ and $u''(\cdot) < 0$.
- $0 < \beta < 1$ is a pure time preference parameter.

The long-run social optimum

- Following Samuelson (1975), we consider a social planner who selects consumptions, capital and fertility in such a way as to maximize the average lifetime welfare at the stationary equilibrium.

- The social planner's problem can be written as follows:

$$\max_{c,d,b,k,n} u(c) + \beta u(d) + \pi\beta^2 u(b)$$

$$\text{s.t. } F\left(k, 1 + \frac{\alpha}{n}\right) = c + \frac{d}{n} + \frac{b\pi}{n^2} + nk$$

The long-run social optimum

- An interior optimum $(c^*, d^*, b^*, k^*, n^*)$ satisfies the following FOCs:

$$
\begin{aligned}
\frac{u'(c^*)}{\beta u'(d^*)} &= \frac{u'(d^*)}{\beta u'(b^*)} = n^* \\
F_k\left(k^*, 1+\frac{\alpha}{n^*}\right) &= n^* \\
k^* + \frac{\alpha F_L\left(k^*, 1+\frac{\alpha}{n^*}\right)}{(n^*)^2} &= \frac{d^*}{(n^*)^2} + \frac{2b^*\pi}{(n^*)^3}
\end{aligned}
$$

The optimum fertility rate

$$\underbrace{k^*}_{\substack{\text{capital dillution} \\ \text{(Solow effect)}}} + \underbrace{\frac{\alpha F_L\left(k^*, 1+\frac{\alpha}{n^*}\right)}{\left(n^*\right)^2}}_{\substack{\text{labor age composition} \\ \text{(Sauvy effect)}}} = \underbrace{\frac{d^*}{\left(n^*\right)^2} + \frac{2b^*\pi}{\left(n^*\right)^3}}_{\substack{\text{intergenerational redistribution} \\ \text{(Samuelson effect)}}}$$

- If $\alpha = 0$, changing fertility does not affect the age composition of the (productive) labor force.
- If $\alpha > 0$: a third effect is at work: labor age-composition (Sauvy).
  - When n grows, this reduces the ratio total labor force / young workers $\frac{N_{t-1}+\alpha N_{t-2}}{N_{t-1}} = \frac{n+\alpha}{n}$.
  - This implies a fall of product per young worker equal to $-\frac{\alpha F_L\left(k^*, 1+\frac{\alpha}{n^*}\right)}{\left(n^*\right)^2}$.

Old workers skills decay and the Sauvy effect

- The fall in product per young worker induced by a higher n depends on the decay α in a non-monotonic way:
    - A rise in α raises the first factor of $\alpha F_L\left(k^*, 1+\frac{\alpha}{n^*}\right)$ but reduces the second factor.
- When the first effect dominates the second one, a larger decay (i.e. a lower α) reduces the negative productivity effect induced by a higher n, leading to a higher optimal fertility.
- The inverse occurs when the second effect dominates the first one.

Back to Sauvy (1962)

- Sauvy (1962) argued that the ageing of the Walloon workforce could lead to a fall of total labor productivity, making a higher fertility rate more desirable for Wallonia.
- This argument can hold in our model through the Sauvy effect.
- Provided $\alpha F_L\left(k^*, 1+\frac{\alpha}{n^*}\right)$ is increasing in α, a higher decay of old workers' skills supports, *ceteris paribus*, a higher fertility.

A few words on corner solutions

- Up to now we assumed that the optimal fertility rate is an interior optimum.
- Interiority of n^* requires (Deardorff 1976, Michel and Pestieau 1993):
  - a sufficiently low degree of substitutability between consumptions at different ages of life
    - Otherwise the Samuelson effect is too weak, leading to $n^* = 0$.
  - a sufficiently low degree of substitutability between capital and labor in the production process.
    - Otherwise the Solow effect is too weak, leading to $n^* = +\infty$.
- Those considerations remain true in the present setting.

Numerical illustrations: data

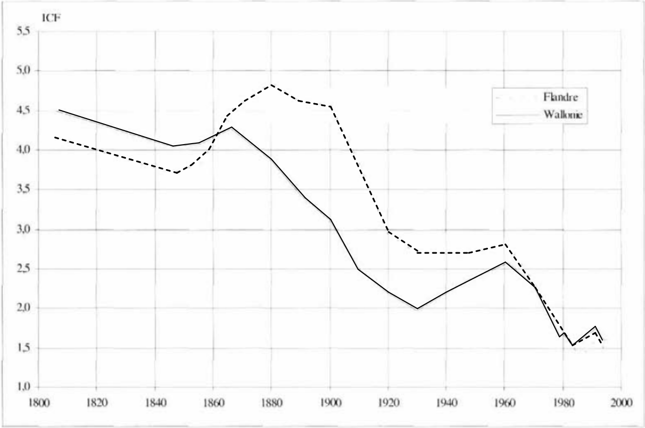

Fig. 1: TFR in Flanders and Wallonia (Source: Capron et al 1998 p. 264).

Numerical illustrations: functional forms

- The temporal utility function takes the standard CIES form:

$$u\left(c_t\right) = \frac{c_t^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}}$$

- The production function takes a CES form:

$$\bar{F}\left(K_t, L_t\right) = A\left[\gamma K_t^{-\rho} + (1-\gamma) L_t^{-\rho}\right]^{-1/\rho}$$

where $A > 0$, $0 < \gamma < 1$, $\rho > -1$, $\rho \neq 0$.

Numerical illustrations: calibration

- Large debates on the relation age/productivity: survey in van Ours and Stoeldraijer (2010).
  - Productivity may be stable or slightly decline after age 50.
- Here we take $\alpha = 1$ as a benchmark and consider the effect of reducing α on optimal fertility.
- Other parameters:

parameters	α	β	σ	π	A	γ	ρ	δ
values	1.00	0.45	1.25	0.48	20	0.49	1.00	1.00

- Sources:
  - β: de la Croix and Michel (2002): quarterly discount factor $= 0.99$.
  - σ: Blundell et al (1994): elasticity of intertemporal substitution $= 1.25$.
  - π: fits life expectancy (1960) $= 69$ years.
  - ρ: de la Croix and Michel (2002): elasticity of substitution between K and $L = 0.50$.
  - γ: de la Croix and Michel (2002): share of labour $= 2/3$.

Numerical illustrations: results

- Corner solution under benchmark calibration! Too high σ.

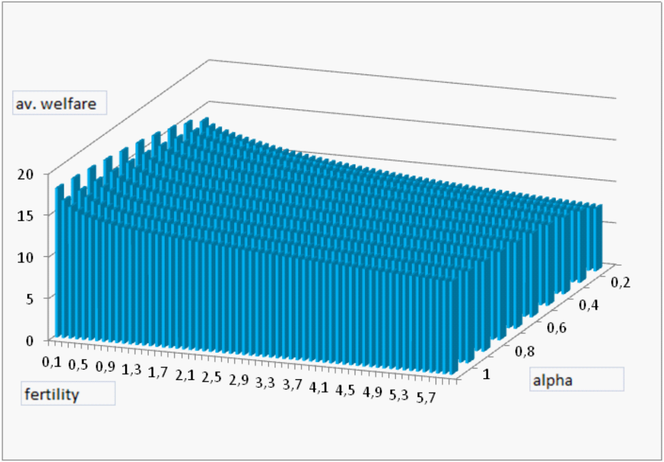

Fig. 2: Average lifetime welfare as a function of fertility n and decay α (benchmark values).

Numerical illustrations: results

- Under $\sigma = 0.5$, n^* goes from 0.5 to 0.6 when α falls from 1 to 0.6. But then goes back to 0.5 as α falls.

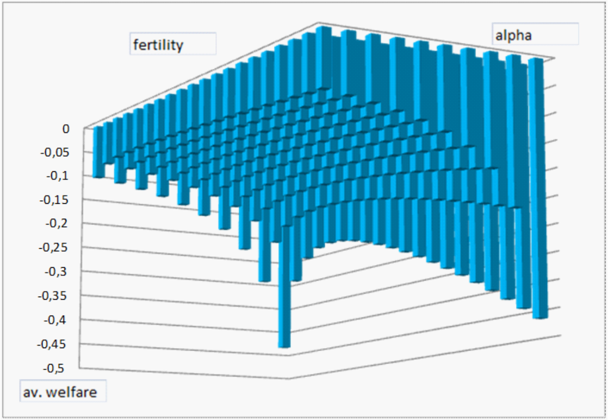

Fig. 3: Average lifetime welfare as a function of fertility (n) and decay (α), under $\sigma = 0.5$.

Numerical illustrations: results

- Finally, under $\sigma = 0.8$ and our current life expectancy ($\pi = 0.95$), n^* is between 0.2 (when $\alpha = 1$) and 0.5 (when $\alpha = 0.3$).

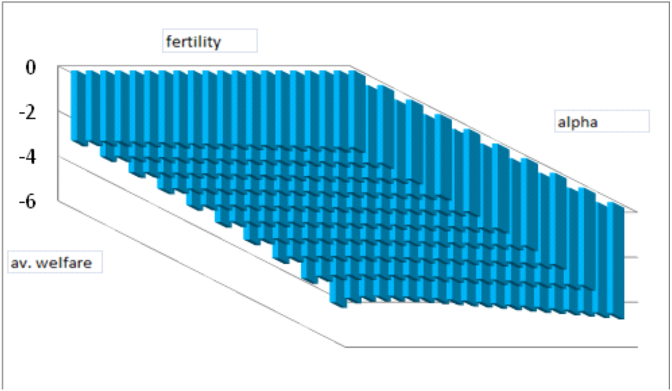

Fig. 4: Average lifetime welfare as a function of fertility (n) and decay (α), under $\sigma = 0.8$ and $\pi = 0.95$.

Conclusions: a contrasted view on Sauvy

- In theory, we can account for Sauvy's views in a Samuelson model.
- But when turning to numbers, even a large decay in old workers' skills does not suffice to support Sauvy's claim.
- Other considerations - culture, knowledge, power - were playing a role in Sauvy's recommendations.
    - Including these remains to be done.